On the First Eigenvalue of the Linearized Operator of the r-th Mean Curvature of a Hypersurface

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Abstract: We generalize Reilly's inequality for the first eigenvalue of immersed submanifolds of $\mathbb{R}^{m+1}$ and the total (squared) mean curvature, to hypersurfaces of $\mathbb{R}^{m+1}$ and the first eigenvalue of the higher order curvatures. We apply this to stability problems. We also consider hypersurfaces in hyperbolic space.

Key words: *Curvatures of hypersurfaces, Reilly's inequality, stability*
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Let M^n be a closed submanifold immersed in $\mathbf{R}^{m+1}$ (i.e., M is compact, connected and $\partial M = \phi$), and let H_1, λ_1 denote the mean curvature and first eigenvalue of the Laplacian of M, respectively. In [R-1], R. Reilly proved:

$$\frac{\lambda_1}{n} \leq \frac{1}{\operatorname{Vol}(M)} \int_M H_1^2 \tag{0.1}$$

and that equality occurs precisely if M is minimally immersed in a sphere of $\mathbf{R}^{m+1}$. In particular, if $n = m$, equality means that M is a sphere.

This result of Reilly can easily be extended to (isometric) immersions in the unit sphere S^{m+1} by applying (0.1) to the immersion of M in $\mathbf{R}^{m+2}$ obtained by $M \to S^{m+1} \subset \mathbf{R}^{m+2}$:

$$\frac{\lambda_1}{n} - 1 \leq \frac{1}{V} \int_M H_1^2, \quad V = \operatorname{Vol}(M). \tag{0.2}$$

For immersions of M^m in the hyperbolic space $\mathbf{H}^{m+1}$, the situation is more subtle. E. Heintze obtained some results [H] (indeed, he considers any Riemannian manifold as an ambient space). The best inequality in $\mathbf{H}^{m+1}$ was finally obtained by A. El Soufi and S. Ilias [S-I]:

$$\frac{\lambda_1}{m} + 1 \leq \frac{1}{V} \int_M H_1^2, \quad m \geq 2, \tag{0.3}$$

and equality occurs precisely if M is minimally immersed in a geodesic sphere of radius $\operatorname{arcsh} \sqrt{\frac{m}{\lambda_1}}$.

A. El Soufi and S. Ilias apply this result to study stable immersions of constant mean curvature. In particular, they obtain the theorems of Barbosa, do Carmo (ambient space $\mathbf{R}^{m+1}$) and Barbosa, do Carmo, Eschenburg (ambient spaces S^{m+1} and $\mathbf{H}^{m+1}$), [B-dC], [B-dC-E]:

The stable hypersurfaces M^m of constant mean curvature, immersed in a space form $\mathbf{R}^{m+1}$, S^{m+1}, or $\mathbf{H}^{m+1}$, are precisely the geodesic hyperspheres.

In this paper we shall continue the study of $\lambda_1(L_r)$, where L_r is the linearized operator of $S_{r+1} = \binom{m}{r+1}H_{r+1}$ arising from normal variations of an immersed hypersurface $M^m \subset N^{m+1}(a)$, $a = 0, \pm 1$, $N^{m+1}(0) = \mathrm{R}^{m+1}$, $N^{m+1}(1) = S^{m+1}$ and $N^{m+1}(-1) = \mathbf{H}^{m+1}$. Here S_r is the r-th elementary symmetric function of the eigenvalues $k_1, \dots, k_n$ of the shape operator A, i.e., $S_0 = 1$, $S_1 = k_1 + \cdots + k_n, \cdots, S_n = k_1 \dots k_n$. We refer the reader to [R-2], [A-dC-C], [Ro] for details concerning L_r. For short, $L_r(f) = \operatorname{div}(T_r \nabla f)$, where T_r is the r-th Newton transformation arising from A; $T_0 = I$, $T_r = S_r I - AT_{r-1}$, (so $L_0 = \Delta$).

In $\mathbf{R}^{m+1}$ we are able to generalize Reilly's result as the best possible one:

$$\lambda_1^{L_r} \int_M H_r \le c(r) \int_M H_{r+1}^2,$$

where M is an immersed hypersurface in $\mathbf{R}^{m+1}$ with $H_{r+1} > 0$, and $c(r) = (m-r)\binom{m}{r}$. Equality holds precisely if M is a sphere.

Using this we generalize the theorems of Barbosa-do Carmo [B-dC] (stability of a constant mean curvature immersion means that M is a sphere) and the theorem of Alencar, do Carmo, and Colares [A-dC-C] (stability of a constant scalar curvature immersion in $\mathbf{R}^{m+1}$ means that M is a sphere). We prove that an immersion of M^n in $\mathbf{R}^{m+1}$ is r-stable if and only if M is a sphere. We shall explain r-stable subsequently; briefly, it means M is a critical point of the functional $\int_M H_r + b\bar{V}(M)$, and the second derivative of this functional at M is nonnegative. Here b is a suitable constant and $\bar{V}(M)$ is the volume bounded by M.

In $\mathbf{H}^{m+1}$ we obtain an extrinsic upper bound for $\lambda_1^{L_r}$, but it is not the best possible one, even for $r = 0$. Consequently, our technique does not yield stability results here. We remark that Alencar, do Carmo and Colares [A-dC-C] have classified the stable constant scalar curvature R immersions of M^m into S^{m+1} as the geodesic spheres, provided $R > 1$, i.e. $H_2 = R - 1 > 0$. For the moment, this is not known to hold in $\mathbf{H}^{m+1}$.

1. Hypersurfaces in $\mathbf{R}^{m+1}$

Let $M = M^m$ be an orientable closed manifold, isometrically immersed in $\mathbf{R}^{m+1}$. We now establish the extrinsic upper bound for λ_1, where λ_1 is the first eigenvalue of the operator L_r, $r = 0, 1, \dots, m-1$.

Theorem 1.1. *If $H_{r+1} > 0$ on M, then*

$$\lambda_1 \int_M H_r \le c(r) \int_M H_{r+1}^2, \quad c(r) = (m-r)\binom{m}{r},$$

and equality holds precisely if M is a sphere of $\mathbf{R}^{m+1}$.

Proof. First observe that L_r is an elliptic operator: by surrounding M by concentric spheres, it is easy to find one strictly convex point of M; then $H_{r+1} > 0$ implies that

L_r is elliptic. (The proof is essentially contained in [K]; cf. also [Ro], p. 20.) Thus, to establish Theorem 1.1, we shall use the minimax characterization of λ_1:

$$\lambda_1 = -\inf\left[\frac{\int_M fL_r(f)}{\int_M f^2}, \int_M f = 0\right]. \tag{1.1}$$

We now proceed with the proof of Theorem 1.1. We may assume that $\int_M X = 0$, where X denotes the position vector of a point of M (just translate M, so that the center of mass is at the origin).

For $1 \le k \le m+1$, let $f = x_k$ be the k-th coordinate function on M; $X = (x_1, \ldots, x_{m+1})$. Then $\int_M f = 0$ and by (1.1) we have

$$\lambda_1 \int_M f^2 \le -\int_M fL_r(f). \tag{1.2}$$

Since $H_{r+1} > 0$ on M, it follows that $H_r > 0$ on M, too ([M-R], Lemma 1). Let n be the unit vector field whose direction is the opposite of the mean curvature vector. It is known (and not difficult) that

$$L_r(X) = -c(r)H_{r+1}n. \tag{1.3}$$

For future reference we state the general formula for $L_r(X)$ in the space form $N^{m+1}(c)$, $c = 0, \pm 1$, ([Ro], Eq. (5.2)).

$$L_r(X) = -c(r)(H_{r+1}n + cH_rX). \tag{1.4}$$

Now we use (1.3) in (1.2) and sum from $k = 1$ to $m+1$ to obtain:

$$\lambda_1 \int_M |X|^2 \le c(r)\int_M H_{r+1}\langle X, n\rangle. \tag{1.5}$$

In $\mathbf{R}^{m+1}$, we have the Minkowski formulae ([Hs], p. 286; our orientation is opposite to that in [Hs]):

$$\int_M (H_r - H_{r+1}\langle X, n\rangle) = 0. \tag{1.6}$$

Thus (1.5) becomes:

$$\lambda_1 \int_M |X|^2 \le c(r)\int_M H_r. \tag{1.7}$$

Now multiply both sides of this equality by $\int_M H_{r+1}^2$ and use Cauchy-Schwarz to obtain:

$$\begin{aligned} c(r)\Big(\int_M H_r\Big)\Big(\int_M H_{r+1}^2\Big) &\ge \lambda_1\Big(\int_M |X|^2\Big)\Big(\int_M H_{r+1}^2\Big) \\ &\ge \lambda_1\Big(\int_M |X|H_{r+1}\Big)^2 \ge \lambda_1\Big(\int_M \langle X, n\rangle H_{r+1}\Big)^2 = \lambda_1\Big(\int_M H_r\Big)^2. \end{aligned}$$

Hence, we have proved:

$$\lambda_1 \int\limits_M H_r \leq c(r) \int\limits_M H_{r+1}^2. \tag{1.8}$$

If equality holds in (1.8), then the equality in our use of Cauchy-Schwarz yields that

$$\Big(\int\limits_M |X| H_{r+1}\Big)^2 = \Big(\int\limits_M \langle X, n\rangle H_{r+1}\Big)^2.$$

By (1.6) and the fact that $H_r > 0$ on M, we conclude that $\int_M \langle X, n\rangle H_{r+1} > 0$, so

$$\int\limits_M |X| H_{r+1} = \int\limits_M \langle X, n\rangle H_{r+1}.$$

Thus $X = kn$ for some function k, hence

$$\nabla_{e_i}(|X|^2) = e_i(\langle X, X\rangle) = 2\langle e_i, X\rangle = 2k\langle e_i, n\rangle = 0.$$

This means the equality is equivalent to $|X| =$ constant, i.e. M is a sphere. □

Corollary 1.2. *In addition to the hypotheses of Theorem* 1.1 *assume that* H_{r+1} *is constant. Then*

$$\lambda_1 \leq c(r) H_{r+1}^{\frac{r+2}{r+1}},$$

and equality holds precisely when M is a sphere.

Proof. We know that $H_r \geq H_{r+1}^{r/r+1}$ and equality means that M is a sphere ([M-R], Lemma 1). Thus, by Theorem 1.1, we have:

$$\lambda_1 \leq c(r) H_{r+1}^2 \frac{\int_M 1}{\int_M H_r} \leq c(r) \frac{H_{r+1}^2}{H_{r+1}^{r/r+1}} = c(r) H_{r+1}^{\frac{r+2}{r+1}}.$$

□

Theorem 1.3. *Let Ω be a compact $m+1$ manifold with $M = \partial\Omega$, and assume Ω to be isometrically immersed in $\mathbf{R}^{m+1}$ so that $H_{r+1} > 0$ on M. Let $\lambda_1 = \lambda_1(L_r)$. Then*

$$\lambda_1 \leq \frac{c(r)}{(m+1)^2} \cdot \frac{V(M)}{V(\Omega)^2} \cdot \int\limits_M H_r,$$

and equality holds precisely when M is a sphere. Here $V(M)$ and $V(\Omega)$ are the volume of M and Ω, respectively.

Proof. Let X (also) denote the pull back of the position vector field to Ω. Then $\operatorname{div}(X) = m+1$ so that

$$\int\limits_\Omega \operatorname{div} X = (m+1)V(\Omega) = \int\limits_M \langle X, n\rangle \leq \int\limits_M |X|. \tag{1.9}$$

Hence,

$$(m+1)^2 V(\Omega)^2 \le \Big(\int\limits_M |X|^2\Big) V(M), \text{ i.e. } \frac{(m+1)^2 V(\Omega)^2}{V(M)} \le \int\limits_M |X|^2. \tag{1.10}$$

Combining this with (1.7) yields Theorem 1.3. Equality in Theorem 1.3 implies:

$$\int\limits_M \langle X, n\rangle = \int\limits_M |X|,$$

and, as before, this means M is sphere. □

2. Stability of Hypersurfaces in R^{m+1}

R. Reilly has calculated the first and second variation of $\int_M H_{r+1}$, for normal variations of hypersurfaces M in a space form $N^{m+1}(c)$. If the normal variation is given by the function f on M, then

$$\frac{\mathrm{d}}{\mathrm{d}t}\Big|_{t=0}\Big(\int\limits_M S_r\Big) = \int\limits_M f\{-(r+1)S_{r+1} + c(m-r+1)S_{r-1}\}, \tag{2.1}$$

where S_j is the j-th symmetric function of M ([R-2], p. 470).

Now let $c = 0$ and $h(X) = \langle X, n(X)\rangle$ be the support function of $M^m \hookrightarrow \mathbf{R}^{m+1}$. It is well-known that

$$\frac{\mathrm{d}}{\mathrm{d}t}\Big|_{t=0}\Big(\int\limits_M h\Big) = \frac{1}{m+1}\int\limits_M f.$$

Thus, for any real number b, we have:

$$\frac{\mathrm{d}}{\mathrm{d}t}\Big(\int\limits_M S_r + bh\Big) = \int\limits_M f\{-(r+1)S_{r+1} + \frac{b}{m+1}\}.$$

Hence, the critical points of the functional $J : M \mapsto \int\limits_M (S_r + bh)$ are those manifolds M for which S_{r+1} is the constant $\frac{b}{(r+1)(m+1)}$.

A stable critical point is a point where $J''(0) \ge 0$ for all normal variations. One can easily compute the second variation of J by using equation (9) of Reilly [R-2] (cf. §2 of [A-dC-C]):

$$J''(0)(f) = \int\limits_M -(r+1)fL_r(f) + [(r+1)c(r+1)H_{r+2} - mc(r)H_1H_{r+1}]f^2. \tag{2.2}$$

Now we can state:

Theorem 2.1. *Let M^m be a closed hypersurface in $\mathbf{R}^{m+1}$ with H_{r+1} constant. Then M is stable if and only if M is a sphere.*

Proof. We apply (2.2) for f being the first eigenfunction of $L_r : L_r(f) + \lambda_1 f = 0$. Assuming that H_{r+1} is constant (hence positive) and M is stable we have:

$$\int_M [(r+1)\lambda_1 + (r+1)c(r+1)H_{r+2} - mc(r)H_1H_{r+1}]f^2 \geq 0. \tag{2.3}$$

By Corollary 1.2 we know: $\lambda_1 f^2 \leq c(r)H_{r+1}^{\frac{r+2}{r+1}} f^2$, hence:

$$\int_M [(r+1)c(r)H_{r+1}^{\frac{r+2}{r+1}} + (r+1)c(r+1)H_{r+2} - mc(r)H_1H_{r+1}]f^2 \geq 0. \tag{2.4}$$

We know that $H_rH_{r+2} \leq H_{r+1}^2$, $r = 0, \ldots, m-2$, with equality at umbilical points ([B-M-V], p. 285, Theorem 1 and p. 288, Remark 3. Furthermore, one can consult [H-L-P], p. 104). By induction, we can see that if $H_{r+1} > 0$, this implies:

$$H_1H_{r+1} \geq H_{r+2},$$

with equality at umbilical points.

Thus, since $(r+1)c(r+1) = (m-r-1)c(r)$,

$$\begin{aligned}
&(r+1)c(r)H_{r+1}^{\frac{r+2}{r+1}} + (r+1)c(r+1)H_{r+2} - mc(r)H_1H_{r+1} \\
&\quad \leq (r+1)c(r)H_{r+1}^{\frac{r+2}{r+1}} + (m-r-1)c(r)H_1H_{r+1} - mc(r)H_1H_{r+1} \\
&\quad = (r+1)c(r)H_{r+1}^{\frac{r+2}{r+1}} - (r+1)c(r)H_1H_{r+1} \\
&\quad = (r+1)c(r)\Big(H_{r+1}^{\frac{r+2}{r+1}} - H_1H_{r+1}\Big).
\end{aligned}$$

Since $H_1 \geq H_{r+1}^{\frac{1}{r+1}}$ with equality at umbilical points ([M-R], Lemma 1), we have $H_1H_{r+1} \geq H_{r+1}^{\frac{r+2}{r+1}}$, hence

$$(r+1)c(r)H_{r+1}^{\frac{r+2}{r+1}} + (r+1)c(r+1)H_{r+2} - mc(r)H_1H_{r+1} \leq 0.$$

By (2.4) we must have equality everywhere, hence M is a sphere. □

3. Hypersurfaces of Hyperbolic Space

Let $M^m \subset H^{m+1}$ be a closed hypersurface and let $X = (\tilde{X}, x) \in \mathbf{R}^{m+2}$, $\tilde{X} \in \mathbf{R}^{m+1}$, $x \in \mathbf{R}^+$, denote a point of H^{m+1} in the Minkowski model. We think of X as the position vector field of M. The metric on $\mathbf{H}^{m+1}$ is that induced by $dx_1^2 + \ldots + dx_{m+1}^2 - dx_{m+2}^2$ and

$$\mathbf{H}^{m+1} = \{(\tilde{X}, x)/|\tilde{X}|^2 - x^2 = -1\}.$$

Here $\tilde{X} = (x_1, \ldots, x_{m+1})$, $x = x_{m+2} > 0$, and $|\tilde{X}|$ is the Euclidean norm on $\mathbf{R}^{m+1}$. It is clear from the definition of the metric that Euclidean isometries T of $\mathbf{R}^{m+1}$, induce isometries of $\mathbf{H}^{m+1}$ by $(\tilde{X}, x) \mapsto (T\tilde{X}, x)$. Thus, after an isometry of $\mathbf{H}^{m+1}$ induced by a translation of $\mathbf{R}^{m+1}$, we can assume the center of gravity of the $\tilde{X}$ coordinate of M to be at 0; i.e., $\int_M x_j = 0$, $1 \leq j \leq m+1$. For fixed j, let $f = x_j$.

Let N be the unit normal vector field to M in $\mathbf{H}^{m+1}$ whose direction is the opposite of the mean curvature vector of M (assuming this vector to be nonzero). Then, if

$L = L_r$ is the linearization of the $r+1$-st symmetric function of curvature of M, we have the vector equation (1.4):

$$L(X) = -c(r)(H_{r+1}N - H_r X), \quad c(r) = (m-r)\binom{m}{r}.$$

This represents $m+2$ equations for the coordinate functions of X and N. We have

$$L(f) = -c(r)(H_{r+1}n_j - H_r x_j).$$

Thus

$$-fL(f) = c(r)(H_{r+1}n_j x_j - H_r x_j^2),$$

and summing up from $j = 1$ to $m+1$:

$$-(m+1)fL(f) = c(r)(H_{r+1}\langle \tilde{N}, \tilde{X}\rangle - H_r|\tilde{X}|^2),$$

where $\tilde{N}$ is the projection of N to $\mathbf{R}^{m+1}$.

Now we proceed as in the Euclidean case to estimate $\lambda_1 = \lambda_1^{L_r}$.

$$\lambda_1 \int_M f^2 \le -\int_M fL(f)$$

becomes

$$\frac{\lambda_1}{c(r)}\int_M |\tilde{X}|^2 \le \int_M (H_{r+1}\langle \tilde{N}, \tilde{X}\rangle - H_r|\tilde{X}|^2).$$

Now $X \cdot X = -1$ and $X \cdot N = 0$, hence $|\tilde{X}|^2 = x^2 - 1$ and $\langle \tilde{X}, \tilde{N}\rangle = xn$, where $n = -e \cdot N$ and $e = (0, \dots, 0, 1) \in \mathbf{R}^{m+2}$. Since X, N and the tangent space to M at X generate the vector space $\mathbf{R}^{n+2}$, the gradient ∇x of x (in the metric on M) can be written as

$$\nabla x = e + (e \cdot X)X - (e \cdot N)N = e - xX + nN$$

(cf. [B-dC-E], p. 131). Thus

$$0 \le |\nabla x|^2 = -1 + x^2 - n^2,$$

hence

$$xn \le \frac{x^2 + n^2}{2} \le x^2 - \frac{1}{2} = |\tilde{X}|^2 + \frac{1}{2}.$$

Assume $H_{r+1} > 0$, so that $H_r \ge H_{r+1}^{\frac{r}{r+1}}$, and let $\bar{H} = \sup H$, $\underline{H} = \inf H$. Then

$$-\int H_r|\tilde{X}|^2 \le -\underline{H}_{r+1}^{\frac{r}{r+1}} \int |\tilde{X}|^2,$$

and

$$\int H_{r+1}\langle \tilde{X}, \tilde{N}\rangle \le \bar{H}_{r+1}\int |\tilde{X}|^2 + \frac{1}{2}\int H_{r+1}.$$

Thus

$$\frac{\lambda_1}{c(r)} \le \bar{H}_{r+1} + \frac{1}{2}\frac{\int H_{r+1}}{\int |\tilde{X}|^2} - \underline{H}_{r+1}^{\frac{r}{r+1}}.$$

We have

$$\int |\tilde{X}|^2 \int H_{r+1}^2 \geq \int n^2 \int H_{r+1}^2 \geq \Big(\int H_{r+1} n\Big)^2 = \Big(\int H_r x\Big)^2 \geq \Big(\int H_r\Big)^2.$$

Here we have used $|\tilde{X}|^2 = x^2 - 1 \geq n^2$ and $\int H_{r+1} n = \int H_r x$ and since L is a divergence operator so $\int L(X) = 0$ by Stoke's theorem. The last inequality follows from $x \geq 1$. Thus we have proved:

Theorem 3.1. *Let M be a closed hypersurface immersed isometrically in $\mathbf{H}^{m+1}$ and suppose $H_{r+1} > 0$ on M. Then*

$$\frac{\lambda_1}{c(r)} \leq \bar{H}_{r+1} + \frac{1}{2}\frac{\bar{H}_{r+1}^3}{\underline{H}_r^2} - \underline{H}_{r+1}^{\frac{r}{r+1}}.$$

If H_{r+1} is constant (necessarily $H_{r+1} > 1$), this becomes:

$$\frac{\lambda_1}{c(r)} \leq H_{r+1} + \frac{1}{2} H_{r+1}^{\frac{r+3}{r+1}} - 1.$$

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