ILL-POSEDNESS FOR THE BENNEY SYSTEM

Adán J. Corcho
Universidade Federal de Alagoas
Departamento de Matemática
Campus A. C. Simões
Tabuleiro do Martins, 57072-970
Maceió-Brasil

Abstract. We discuss ill-posedness issues for the initial value problem associated to the Benney system. To prove our results we use the method introduced by Kenig, Ponce and Vega [10] to show ill-posedness for some canonical dispersive equations.

1. Introduction

We consider the Initial Value Problem (IVP) associated to the Benney system, that is,

$$\begin{cases} i\partial_t u + \partial_x^2 u = \alpha u\eta + \beta |u|^2 u, \quad t, x \in \mathbb{R}, \\ \partial_t \eta + \lambda \partial_x \eta = \gamma \partial_x |u|^2, \\ u(x,0) = u_0, \quad \eta(x,0) = \eta_0, \end{cases} \tag{1.1}$$

where u is a complex valued function, η is a real valued function, $\lambda = \pm 1$ and α, β and γ are real constants.

This system appears in general theory of water wave interaction in a nonlinear medium and was introduced by Benney [3, 4]. The solvability of the system (1.1) has been studied by several authors. Yajima and Oikawa [16] applied the inverse scattering method and found N-soliton solutions of (1.1) when $\lambda = 1$, $\gamma = -1$ and $\beta = 0$. Ma [12] proposed a simpler approach of the inverse scattering method. Laurençot [11] considered the orbital stability for a weak solution in $H^1(\mathbb{R})$ with $\beta = 0$. Tsutsumi and Hatano [14] showed local well-posedness for a resonant case ($\lambda = 0$) in $H^{k+1/2}(\mathbb{R}) \times H^k(\mathbb{R})$ with $k = 0$ when $\beta = 0$ and with $k \in \mathbb{Z}^+$ when $\beta \neq 0$. They also obtained global well-posedness in similar spaces for $\lambda = 0$ and $\alpha = \gamma = 1$ via the conservation laws

$$I_1(t) = \int_{-\infty}^{+\infty} |u(x,t)|^2 dx = I_1(0), \tag{1.2}$$

$$I_2(t) = \int_{-\infty}^{+\infty} \left(\eta(x,t)|u(x,t)|^2 + |u_x(x,t)|^2 + \tfrac{\beta}{2}|u(x,t)|^4 \right) dx = I_2(0), \tag{1.3}$$

and

$$I_3(t) = \int_{-\infty}^{+\infty} \left(\eta^2(x,t) + 2Im(u(x,t)\overline{u_x(x,t)} \right) dx = I_3(0). \tag{1.4}$$

Moreover, using a Gauge transformation they also extended these results to the case when the system is not necessarily resonant [15]. Bekiranov, Ogawa and

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Ponce [1] showed well-posedness for initial data $(u_0, \eta_0) \in H^k(\mathbb{R}) \times H^{k-1/2+\epsilon}(\mathbb{R})$ with $1/2 \leq k < 1$ and $\epsilon > 0$ when $\beta \neq 0$ and $(u_0, \eta_0) \in H^k(\mathbb{R}) \times L^{1/k}(\mathbb{R})$ with $0 < k < 1/2$ when $\beta = 0$. The best local well-posedness result for the IVP (1.1) is in $L^2(\mathbb{R}) \times H^{-1/2}(\mathbb{R})$, proved recently by Ginibre, Tsutsumi and Velo [9] and Bekiranov, Ogawa and Ponce [2].

In this work, we discuss some ill-posedness issues regarding this system in the focusing case[1]. In this case, our results show that the best local well-posedness result, in Sobolev spaces, is for data in $L^2(\mathbb{R}) \times H^{-1/2}(\mathbb{R})$ as was suggested by Bekiranov, Ogawa and Ponce in [2]. The proof of these results is based on the ideas used by Kenig, Ponce and Vega [10] to show ill-posedness for the nonlinear Schrödinger, Korteweg de Vries and modified Korteweg-de Vries equations (see also Biagioni and Linares [6, 7]). The notion of local well-posedness used in the proof of the above results includes: existence, uniqueness and persistence property of the solution in certain time interval and instead of continuous dependence of the solution upon data we will require that the data-solution mapping $(u_0, \eta_0) \longmapsto (u(t), \eta(t))$, be uniformly continuous, where $(u(t), \eta(t))$ is the solution associated to the IVP (1.1) with initial data $(u_0, \eta_0) \in H^k(\mathbb{R}) \times H^l(\mathbb{R})$ and $\|(u_0, \eta_0)\|_{H^k \times H^l} \leq C_0$. In the case when any one of the requirements in the notion of local well-podedness fails, we say that the IVP (1.1) is ill-posed.

The following result is due to Ginibre, Tsutsumi and Velo [9].

Theorem 1.1. *The Benney System (1.1) for initial data $(u_0, \eta_0) \in H^k(\mathbb{R}) \times H^l(\mathbb{R})$ is locally well-posed provided*

$$-1/2 < k - l \leq 1 \quad \textit{and} \quad 0 \leq l + 1/2 \leq 2k. \tag{1.5}$$

The solution satisfies:

$$u \in C([0,T]; H^k(\mathbb{R})), \quad \eta \in C([0,T]; H^l(\mathbb{R})). \tag{1.6}$$

Note that if $k - l$ is fixed the lowest allowed values of (k, l) are attained for $k - l = \frac{1}{2}$ and are given by $(k, l) = (0, -1/2)$. Moreover, local well-posedness was shown by Bekiranov, Ogawa and Ponce [2] in the line $l = k - 1/2$ with $k \geq 0$.

In both works due to Ginibre, Tsutsumi and Velo [9] and Bekiranov, Ogawa and Ponce [2], the best result obtained for local well-posedness for the IVP (1.1) is in the space $L^2(\mathbb{R}) \times H^{-1/2}(\mathbb{R})$. Since scaling argument cannot be applied to the Benney system to obtain a criticality notion it is not clear whether this result is optimal. Here we show that this result is in fact the best possible to get local well-posedness. For this, we prove the following theorem concerning ill-posedness for the IVP (1.1).

Theorem 1.2. *The Benney System (1.1) is ill-posed in $H^k(\mathbb{R}) \times H^l(\mathbb{R})$ for $\beta < 0$ provided*

$$-1/3 \leq k < 0 \quad \textit{and} \quad k(2l + 3) + 1 \geq 0. \tag{1.7}$$

To prove Theorem 1.2 we will follow closely [10]. The main ingredient in our proof is the use of the properties of the solitary wave solutions of the system (1.1). The existence of such special functions for any speed of propagation, $c > 0$, and the exponential decay are strongly applied.

Remark 1.3. *For $\beta \geq 0$, it is not possible to employ the same argument used in the proof of Theorem 1.2. Nevertheless, we can give a criticality notion for the special*

[1] Similar to the theory of the cubic NLS we say the Benney system is "focusing" in the case $\beta < 0$.

case $\beta = \lambda = 0$. *Indeed, if* (u, η) *is a solution of the system (1.1) with initial data* $(u_0(x), \eta_0(x))$ *then*

$$u_\mu(x,t) = \mu^{3/2} u(\mu x, \mu^2 t),$$
$$\eta_\mu(x,t) = \mu^2 \eta(\mu x, \mu^2 t),$$

solves (1.1) with initial data $u_{\mu 0} = \mu^{3/2} u_0(\mu x)$ *and* $\eta_{\mu 0} = \mu^2 \eta_0(\mu x)$. *Now taking the homogeneous derivative of order* k *in* L^2 *for* u_μ *and* l *in* L^2 *for* η_μ*, we obtain the followings*

$$\|D_x^k u_\mu\|_{L^2}^2 = \mu^{2+2k} \|D_x^k u\|_{L^2}^2,$$
$$\|D_x^l \eta_\mu\|_{L^2}^2 = \mu^{3+2l} \|D_x^l \eta\|_{L^2}^2.$$

Hence, the notion of criticality is well defined for the Benney system with initial data $(u_0, \eta_0) \in H^k(\mathbb{R}) \times H^l(\mathbb{R})$*, and the critical values turn out to be* $k = -1$ *and* $l = -3/2$. *We note that the optimal relation between* k *and* l *is* $k - l = 1/2$.

In Figure 1 we compare the results for local well-posedness given by Theorem 1.1 with our results for ill-posedness in Theorem 1.2.

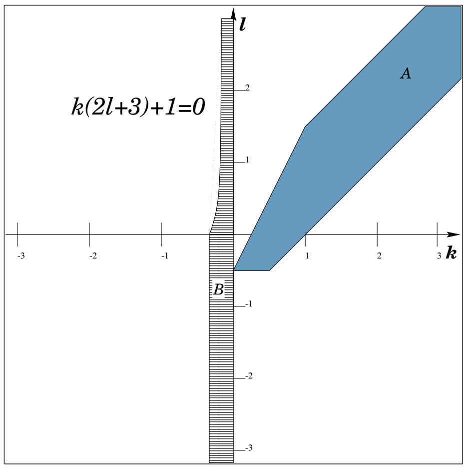

FIGURE 1. The region A contains indices (k,l) where local well-posedness was shown in [2, 9], and the region B contains those where ill-posedness is shown by our example.

2. Solitary waves

In this section we obtain solitary wave solutions for the Benney system.

We will look for solutions of equation (1.1) of the form:

$$u(x,t) = e^{i\omega t}\phi(x - ct) \quad \text{and} \quad \eta(x,t) = \psi(x - ct), \tag{2.8}$$

where $\omega > 0$, $c > 0$ and ϕ and ψ are two smooth L^2-functions which decrease rapidly to zero at infinity (see [11]).

Substituting (2.8) in (1.1) we have the following system of ordinary differential equations for ϕ and ψ

$$\begin{cases} -ic\phi' - \omega\phi + \phi'' = \alpha\phi\psi + \beta|\phi|^2\phi \\ (\lambda - c)\psi' = \gamma(|\phi|^2)'. \end{cases} \tag{2.9}$$

Taking $c > |\lambda| = 1$, we obtain

$$-ic\phi' - \omega\phi + \phi'' = (\beta + \frac{\alpha\gamma}{\lambda - c})|\phi|^2\phi. \tag{2.10}$$

Setting $\phi(x) = e^{\frac{icx}{2}} h(x)$, where h is a real valued function and using (2.10) we have

$$h'' - (\omega - \frac{c^2}{4})h - (\beta - \frac{\alpha\gamma}{c - \lambda})h^3 = 0. \tag{2.11}$$

We can see [5] and [13] for the following statements. The equation (2.11) has positive, even, smooth and exponentially decreasing solutions if the conditions

$$\omega - \frac{c^2}{4} > 0 \quad \text{and} \quad \beta(c - \lambda) - \alpha\gamma < 0, \tag{2.12}$$

are satisfied. The solution in this case is given by

$$h(x) = \frac{2\mu\sigma}{e^{-\sigma x} + e^{\sigma x}} = \mu\sigma \operatorname{sech}(\sigma x), \tag{2.13}$$

where

$$\mu = \sqrt{\frac{2(c - \lambda)}{\alpha\gamma - \beta(c - \lambda)}} \quad \text{and} \quad \sigma = \sqrt{\omega - \frac{c^2}{4}}. \tag{2.14}$$

The set of non trivial solutions of (2.11) in $H^1(\mathbb{R})$ is empty if the condition (2.12) fails.

Remark 2.1. *For $c > 1$ and $\omega > \frac{c^2}{4}$, the condition (2.12) holds in the following cases:*

(i) $\beta < 0, \quad c > \max\left\{1, \lambda + \frac{\alpha\gamma}{\beta}\right\}$.
(ii) $\beta = 0, \quad \alpha\gamma > 0$.
(iii) $\beta > 0, \quad 1 < c < \lambda + \frac{\alpha\gamma}{\beta}$.

We are interested in the case (i). Here, the speed of propagation ($c > 1$) is not restricted to a bounded interval and this fact is strongly used in our argument.

Finally, we have the following expressions for the solitary waves:

$$\begin{cases} u_{c,\omega}(x,t) = e^{i\omega t} e^{\frac{ic}{2}(x-ct)} \mu g_\sigma(x - ct), \\ \\ \eta_{c,\omega}(x,t) = -\frac{2\gamma}{\alpha\gamma - \beta(c-\lambda)} g_\sigma^2(x - ct), \\ \\ g_\sigma(x) := \sigma g(\sigma x), \ g(x) = \operatorname{sech}(x). \end{cases} \tag{2.15}$$

3. Proof of Theorem 1.2

The idea of the proof is the following: we will take two solitary waves as in (2.15) as our initial data. We will see that under some assumptions they will remain close at initial time and then we will see the evolution of the solutions associated to them to find a contradiction.

Without loss of generality, we may assume $\beta = -1$ and $\alpha = \gamma = \lambda = 1$ in (1.1).

Taking

$$N \gg 1, \quad c = 2N \quad \text{and} \quad \omega = N^2 + \sigma^2, \tag{3.16}$$

and using (2.15) we have that the pair

$$\begin{cases} u_{\sigma,N}(x,t) = e^{-it(N^2-\sigma^2)} e^{iNx} \mu(N) g_\sigma(x - 2tN), \\ \eta_{\sigma,N}(x,t) = -\frac{1}{N} g_\sigma^2(x - 2tN), \\ \mu(N) = \sqrt{\frac{2N-1}{N}}, \end{cases} \tag{3.17}$$

is the solution of the Benney system (1.1) with initial data $(e^{iNx}\mu(N)g_\sigma(x), -\frac{1}{N}g_\sigma^2(x))$.

Taking Fourier transform we have

$$\widehat{u}_{\sigma,N}(\xi,t) = e^{it(N^2+\sigma^2-2N\xi)} \mu(N) \widehat{g}\Big(\frac{\xi - N}{\sigma}\Big) \tag{3.18}$$

and

$$\widehat{\eta}_{\sigma,N}(\xi,t) = -\frac{\sigma}{N} e^{-2itN\xi} \widehat{g^2}\Big(\frac{\xi}{\sigma}\Big). \tag{3.19}$$

Let us set

$$N_j \simeq N, \quad N_1 < N_2, \quad \omega_j = N_j^2 + \sigma^2, \quad j = 1,2 \tag{3.20}$$

and write

$$u_j(x,t) := u_{\sigma,N_j}(x,t) \quad \text{and} \quad \eta_j := \eta_{\sigma,N_j}(x,t). \tag{3.21}$$

The fundamental theorem of calculus and the mean value theorem yield the following inequalities

$$\begin{aligned} |\widehat{u_1}(\xi,0) - \widehat{u_2}(\xi,0)|^2 &= \Big|\mu(N_1)\widehat{g}\Big(\frac{\xi - N_1}{\sigma}\Big) - \mu(N_2) g\Big(\frac{\xi - N_2}{\sigma}\Big)\Big|^2 \\ &\lesssim \mu^2(N_1)\Big|\widehat{g}\Big(\frac{\xi - N_1}{\sigma}\Big) - \widehat{g}\Big(\frac{\xi - N_2}{\sigma}\Big)\Big|^2 \\ &\quad + |\mu(N_1) - \mu(N_2)|^2 \Big|\widehat{g}\Big(\frac{\xi - N_2}{\sigma}\Big)\Big|^2 \\ &\simeq \Big|\int_0^1 \widehat{g}\,'\Big(\frac{\xi - N_2 + t(N_2 - N_1)}{\sigma}\Big)\Big(\frac{N_2 - N_1}{\sigma}\Big) dt\Big|^2 \\ &\quad + |\mu(N_1) - \mu(N_2)|^2 \Big|\widehat{g}\Big(\frac{\xi - N_2}{\sigma}\Big)\Big|^2 \\ &\le |N_2 - N_1|^2 \sigma^{-2} \Big(\int_0^1 \Big|\widehat{g}\,'\Big(\frac{\xi - N_2 + t(N_2 - N_1)}{\sigma}\Big)\Big| dt\Big)^2 \\ &\quad + |\mu'(N_0)|^2 |N_1 - N_2|^2 \Big|\widehat{g}\Big(\frac{\xi - N_2}{\sigma}\Big)\Big|^2, \end{aligned}$$

with $N_0 \in (N_1, N_2)$, $\mu'(N_0) = \frac{1}{2N_0^{\frac{3}{2}}\sqrt{2N_0 - 1}} \simeq \frac{1}{N^2}$. Hence

$$\|u_1(\cdot,0) - u_2(\cdot,0)\|_k^2 \lesssim |N_1 - N_2|^2 \sigma^{-2} I_1 + \frac{|N_1 - N_2|^2}{N^4} I_2, \tag{3.22}$$

where

$$I_1 = \int (1 + |\xi|^2)^k \Big(\int_0^1 \Big|\widehat{g}\,'\Big(\frac{\xi - N_2 + t(N_2 - N_1)}{\sigma}\Big)\Big| dt\Big)^2 d\xi$$

and

$$I_2 = \int (1 + |\xi|^2)^k \Big|\widehat{g}\Big(\frac{\xi - N_2}{\sigma}\Big)\Big|^2 d\xi.$$

Let

$$\sigma = N^{-2k}. \tag{3.23}$$

Taking $k > -\frac{1}{2}$ $(N^{-2k} < N)$, $\xi \in B_\sigma(tN_1 + (1-t)N_2)$ then $|\xi| \simeq N$ for $t \in [0,1]$ and using that $\widehat{g} \in S(\mathbb{R})$ and $\widehat{g}$ concentrates in $B_1(0)$ we have the following estimates for I_1 and I_2.

$$\begin{aligned}
I_1 &\le \int (1+|\xi|^2)^k \Big(\int_0^1 \Big|\widehat{g}\,'\Big(\frac{\xi - N_2 + t(N_2-N_1)}{\sigma}\Big)\Big|^2 dt\Big) d\xi \\
&= \int_0^1 \int (1+|\xi|^2)^k \Big|\widehat{g}\,'\Big(\frac{\xi - N_2 + t(N_2-N_1)}{\sigma}\Big)\Big|^2 d\xi dt \\
&\le CN^{2k}\sigma \int_0^1 \int \Big|\widehat{g}\,'\Big(y - \frac{tN_1 + (1-t)N_2}{\sigma}\Big)\Big|^2 dy dt \\
&\le C\|\widehat{g}\,'\|_{L^2}^2
\end{aligned}$$

and

$$\begin{aligned}
I_2 &\simeq N^{2k} \int \Big|\widehat{g}\Big(\frac{\xi - N_2}{\sigma}\Big)\Big|^2 d\xi \\
&= N^{2k}\sigma\|\widehat{g}\|_{L^2}^2 = \|g\|_{L^2}^2.
\end{aligned}$$

Using (3.22) and the estimates above it follows that

$$\|u_1(\cdot,0) - u_2(\cdot,0)\|_k^2 \lesssim |N_1 - N_2|^2 N^{4k} + \frac{|N_1 - N_2|^2}{N^4}. \tag{3.24}$$

Now we consider the solutions $u_j(x,t)$, $j = 1,2$, at time $t = T$. Observe that

$$\|u_j(\cdot,T)\|_s^2 = \|u_j(\cdot,0)\|_s^2 \simeq N^{2s}\sigma\|g\|_{L^2}^2, \quad s \in \mathbb{R}, \quad j = 1,2. \tag{3.25}$$

If $s = k$ then (3.25) gives

$$\|u_j(\cdot,T)\|_k^2 \simeq \|g\|_{L^2}^2. \tag{3.26}$$

On the the other hand, the frequencies of $u_j(\cdot,T)$, $j = 1,2$, are localized in $B^* = B_\sigma(N_1) \cup B_\sigma(N_2)$. Hence $|\xi| \simeq N$ and consequently

$$\|u_1(\cdot,T) - u_2(\cdot,T)\|_k^2 \simeq N^{2k}\|u_1(\cdot,T) - u_2(\cdot,T)\|_{L^2}^2. \tag{3.27}$$

Now, $u_j(\cdot,T)$ concentrates in $B_{\sigma^{-1}}(2TN_j)$, $j = 1,2$. Therefore, for given $T > 0$, we take N_1 and N_2 such that

$$T|N_1 - N_2| \gg \sigma^{-1} = N^{2k}. \tag{3.28}$$

We have that there is no interaction of u_j, $j = 1,2$, at time $t = T$; hence using (3.25) with $s = 0$ we obtain

$$\|u_1(\cdot,T) - u_2(\cdot,T)\|_{L^2}^2 \simeq \|u_1(\cdot,T)\|_{L^2}^2 + \|u_2(\cdot,T)\|_{L^2}^2 \simeq \sigma. \tag{3.29}$$

Combining (3.27) and (3.29) we obtain

$$\|u_1(\cdot,T) - u_2(\cdot,T)\|_k^2 \ge CN^{2k}\sigma = C. \tag{3.30}$$

Taking

$$N_1 = N \quad \text{and} \quad N_2 = N + \delta N^{-2k} \quad \text{with} \quad \delta > 0, \tag{3.31}$$

we get from (3.24)

$$\|u_1(\cdot,0) - u_2(\cdot,0)\|_k^2 \le C\delta^2(1 + N^{-4(k+1)}) \le C\delta^2. \tag{3.32}$$

Here we have used that $k > -\frac{1}{2}$.

Since $k<0$, given $\delta,\ T>0$, we can take N so large that

$$T|N_1-N_2| = T\delta N^{-2k} \gg N^{2k} \iff N^{-4k} \gg \frac{1}{T\delta}, \tag{3.33}$$

and hence (3.28), (3.29) and (3.30) hold.

The initial data $\eta_j(x,0),\ j=1,2,$ satisfy

$$\begin{aligned}
\|\eta_j(\cdot,0)\|_l^2 &= \frac{\sigma^2}{N_j^2}\int (1+|\xi|^2)^l |\widehat{g^2}(\frac{\xi}{\sigma})|^2 d\xi \\
&= \frac{\sigma^3}{N_j^2}\int (1+\sigma^2 y^2)^l |\widehat{g^2}(y)|^2 dy \\
&\simeq \frac{\sigma^{3+2l}}{N^2}\int (N^{4k}+y^2)^l |\widehat{g^2}(y)|^2 dy \\
&\leq N^{-2(k(2l+3)+1)} \begin{cases} \|g^2\|_l^2, & l\geq 0 \\ N^{4kl}\|g^2\|_{L^2}^2, & l<0 \end{cases} \\
&\leq C,
\end{aligned}$$

whenever

$$k(2l+3)+1 \geq 0, \text{ for } l\geq 0 \quad \text{and} \quad k\geq -\frac{1}{3}, \text{ for } l<0. \tag{3.34}$$

On the other hand,

$$\begin{aligned}
\|\eta_1(\cdot,0)-\eta_2(\cdot,0)\|_l^2 &= \sigma^2\Big(\frac{1}{N_1}-\frac{1}{N_2}\Big)^2 \int (1+|\xi|^2)^l |\widehat{g^2}(\frac{\xi}{\sigma})|^2 d\xi \\
&\simeq \frac{\sigma^3}{N^4}(N_1-N_2)^2 \int (1+\sigma^2 y^2)^l |\widehat{g^2}(y)|^2 dy \\
&= \frac{\sigma^{3+2l}}{N^4} N^{-4k}\delta^2 \int (N^{4k}+y^2)^l |\widehat{g^2}(y)|^2 dy \\
&\leq \delta^2 N^{-2(k(2l+5)+2)} \begin{cases} \|g^2\|_l^2, & l\geq 0 \\ N^{4kl}\|g^2\|_{L^2}^2, & l<0 \end{cases} \\
&\leq C\delta^2,
\end{aligned}$$

where in the last inequality we have used

$$k(2l+5)+2 \geq 0, \text{ for } l\geq 0 \quad \text{and} \quad k\geq -\frac{2}{5}, \text{ for } l<0. \tag{3.35}$$

Note that for $k<0$ the condition (3.34) implies the condition (3.35). This completes the proof of Theorem 1.2.

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E-mail address: adan@mat.ufal.br